# Weekly Review #7

1/19/10

① +½ ⓐ $\lim_{x\to 1^+} \begin{cases} x^2-2, & x<1 \\ -\frac{1}{2}x+1, & x\geq 1 \end{cases}$ use $-\frac{1}{2}x+1 \to -\frac{1}{2}(1)+1 = \boxed{+\frac{1}{2}}$

+½ ⓑ $\lim_{x\to\infty} \frac{6x+1}{|6-2x|}$ as $x\to\infty$ the constants disappear in significance and both top and bottom are positive

so $\to \frac{6x}{2x} \Rightarrow \boxed{3}$

+½ ⓒ $\lim_{x\to 5} \frac{5-6x+x^2}{5-x} \Rightarrow \lim_{x\to 5} \frac{(x-5)(x-1)}{5-x} \Rightarrow -\frac{\cancel{(x-5)}(x-1)}{\cancel{(x-5)}} \Rightarrow 1-x \Rightarrow 1-5$

$\boxed{=-4}$

+½ ⓓ $\lim_{x\to 3} 5-2x+x^2 \Rightarrow 5-2(3)+(3)^2 \Rightarrow \boxed{8}$

2. a) $f(x) = x\csc(x)$ at $x = \frac{\pi}{6}$

+1

$f'(x) = x \cdot -\csc(x)\cot(x) + \csc(x)\cdot 1 \Rightarrow \csc(x)(1 - x\cot(x))$

$f'(\frac{\pi}{6}) = \csc(\frac{\pi}{6})(1 - \frac{\pi}{6}(\cot(\frac{\pi}{6}))) \Rightarrow 2(1 - \frac{\pi}{6}(\sqrt{3})) = \boxed{2 - \frac{\pi\sqrt{3}}{3}}$

slope $\approx 0.186$

orig. pt. $f(\frac{\pi}{6}) = \frac{\pi}{6}\csc(\frac{\pi}{6}) \Rightarrow \frac{\pi}{3}$ $(\frac{\pi}{6}, \frac{\pi}{3})$

$y = m(x - x_1) + y_1$

tangent $\Rightarrow$ $\boxed{L(x) = 2 - \frac{\pi\sqrt{3}}{3}(x - \frac{\pi}{6}) + \frac{\pi}{3}}$

b) $f(x) = \begin{cases} x^2 - 2, & x < 1 \\ -\frac{1}{2}x - \frac{5}{2}, & x \geq 1 \end{cases}$

$\lim_{x \to 1^-} f(x) = -1$

$\lim_{x \to 1^+} f(x) = -2$

no limit at $x = 1$ so no tangent line

+1

3 (a) $xy = \sin x + y^2 \rightarrow x\frac{\partial y}{\partial x} + y\frac{\partial x}{\partial x} = \cos x\frac{\partial x}{\partial x} + 2y\frac{\partial y}{\partial x}$

$\frac{\partial y}{\partial x}(x-2y) = \cos x - y \qquad \boxed{\frac{\partial y}{\partial x} = \frac{\cos x - y}{(x-2y)}}$ $+\frac{1}{2}$

(b) $y = \sin(x^3 - 5x + 1) \rightarrow \boxed{\cos(x^3-5x+1)\cdot(3x^2-5)}$ $+\frac{1}{2}$

(c) $y = (x^3-1)\cos x \rightarrow (x^3-1)\cdot -\sin x + \cos x \cdot 3x^2$

$\boxed{\frac{\partial y}{\partial x} = 3x^2\cos x - (x^3-1)\sin x}$ $+\frac{1}{2}$

(d) $y = \frac{2x+5}{3x-1} \Rightarrow \frac{(3x-1)(2) - (2x+5)(3)}{(3x-1)^2} \Rightarrow \frac{\cancel{6x} - 2 - \cancel{6x} - 15}{(3x-1)^2}$

$\boxed{\frac{\partial y}{\partial x} = \frac{-17}{(3x-1)^2}}$ $+\frac{1}{2}$

(4) $f'(x) = 4x^3 - 8x^2$

$0 = 4x^2(x-2)$

$f'(x) = 0$ at $0, 2$

$-$ | $-$ | $+$

0    2    increasing

decreasing   min

$f''(x) = 12x^2 - 16x$

$0 = 4x(3x-4)$

$f''(x) = 0$ at $x = 0, \frac{4}{3}$

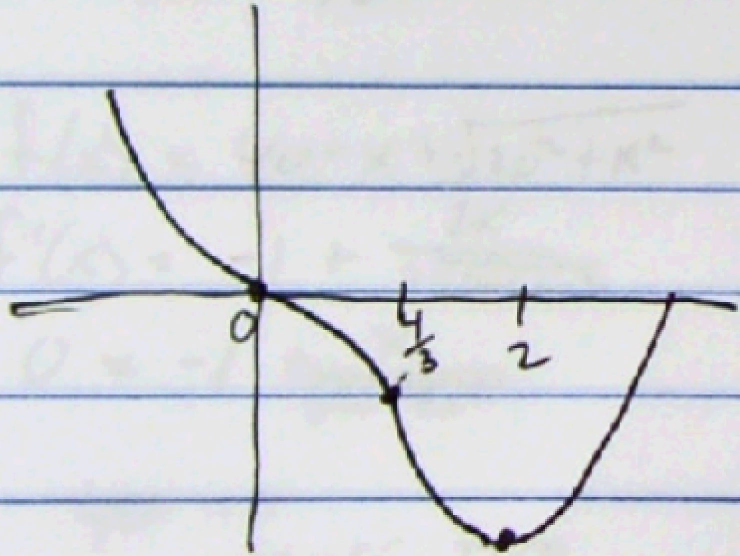
$+$ | $-$ | $+$

up   0   down   $\frac{4}{3}$   up

inflect. pt.

0 - Blank
$\frac{1}{2}$ - D
1 - C
$1\frac{1}{2}$ - B
2 - A

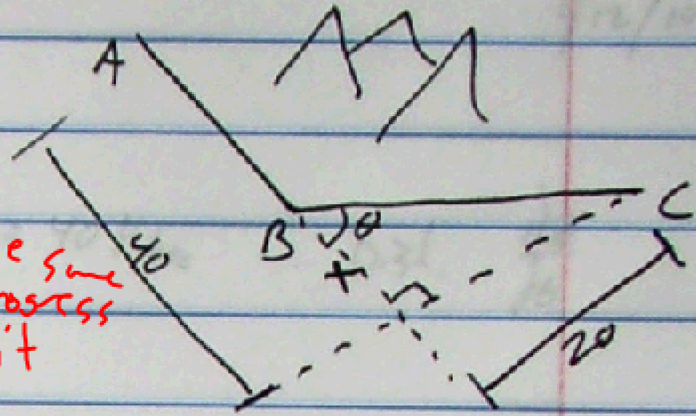
2 pts




possible graph

(5) Total Dist = AB + BC

(a)




$AB = 40 - x \quad , \quad x = \dfrac{20}{\tan\theta}$

$\frac{1}{2}$-pt - made same process

1pt → got it

$BC = \dfrac{20}{\sin\theta}$

$$\text{Total} = \underbrace{40 - \frac{20}{\tan\theta}}_{AB} + \underbrace{\frac{20}{\sin\theta}}_{BC} \rightarrow 40 - \frac{20\sin\theta - 20\tan\theta}{\tan\theta\sin\theta}$$

$$= 40 - 20\left(\frac{\sin\theta - \tan\theta}{\frac{\sin^2\theta}{\cos\theta}}\right) \rightarrow 40 - 20\left(\frac{\cancel{\sin\theta}}{1}\cdot\frac{\cos\theta}{\sin^{\cancel{2}}\theta} - \frac{\cancel{\sin\theta}}{\cancel{\cos\theta}}\cdot\frac{\cancel{\cos\theta}}{\sin^{\cancel{2}}\theta}\right)$$

$$= 40 - 20\left(\frac{\cos\theta - 1}{\sin\theta}\right)$$

but $\tan\frac{\theta}{2} = \dfrac{1-\cos\theta}{\sin\theta}$

so factor out a neg.

$$= 40 + 20\left(\frac{1-\cos\theta}{\sin\theta}\right) \Rightarrow \boxed{40 + 20\tan\frac{\theta}{2}}$$

(b) $\text{Dist} = 40 + 20\tan\frac{1}{2}\theta$ or : $\text{Dist} = 40 - x + \sqrt{20^2 + x^2}$

$50 = 40 + 20\tan\frac{1}{2}\theta$

$\frac{1}{2} = \tan\frac{1}{2}\theta$

$\theta = 0.927$

$x = \frac{20}{\tan\theta}$, $\boxed{x = 15}$

$50 = 40 - x + \sqrt{20^2 + x^2}$

$10 + x = \sqrt{20^2 + x^2}$

$\cancel{x^2} + 20x + 100 = 400 + \cancel{x^2}$

$\boxed{x = 15}$ or 25

(c) $f(x) = 40 - x + \sqrt{20^2 + x^2}$ (use this one b/c it has the restriction)

$f'(x) = -1 + \frac{2x}{2\sqrt{20^2 + x^2}}$

$0 = -1 + \frac{x}{\sqrt{20^2 + x^2}}$

$400 = 0$

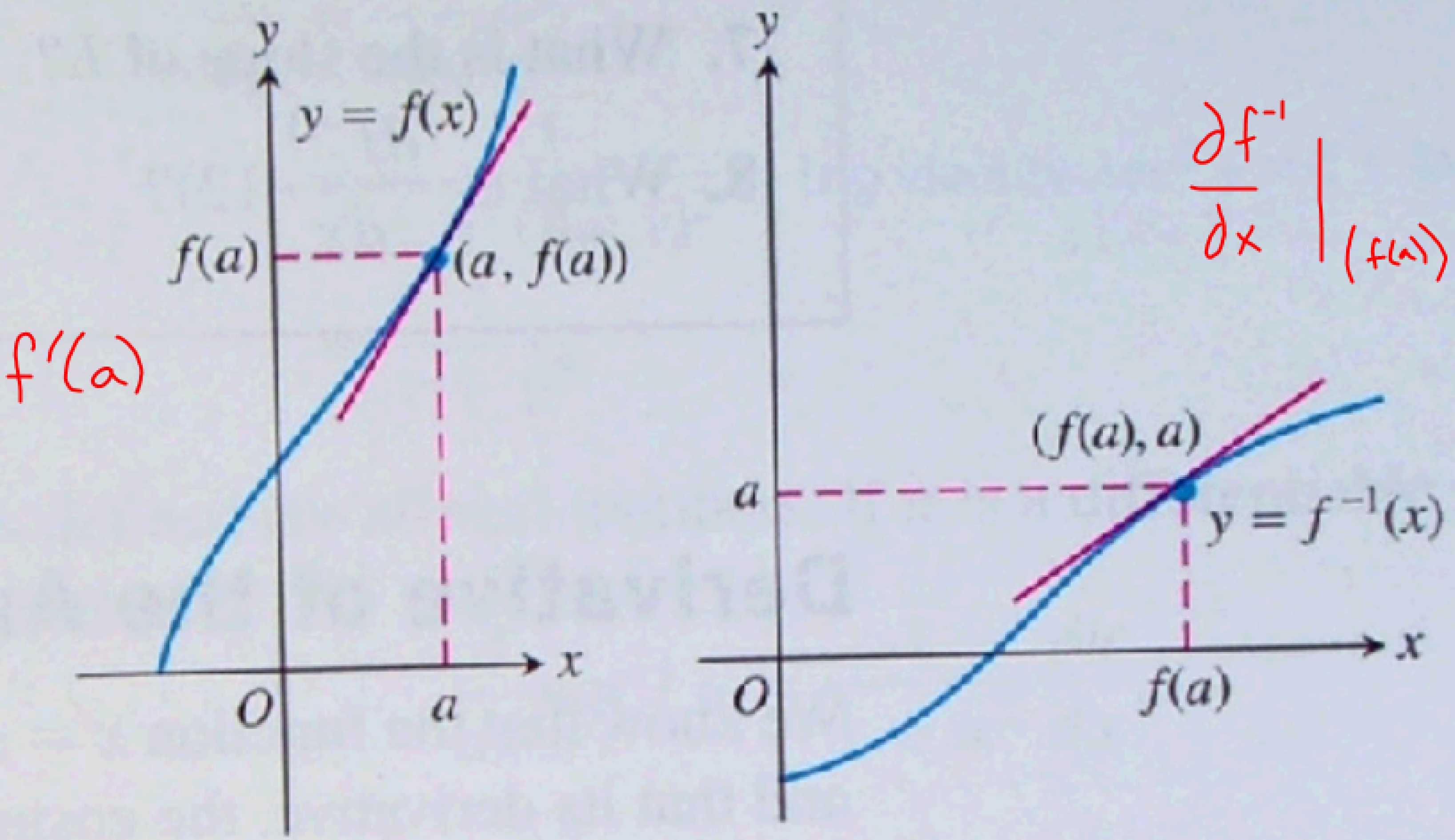
Never zero

but undefined when $x = 20$

$\theta = 45°$

1 pt - got both

$\frac{1}{2}$ pt - about $\frac{1}{2}$




The slopes are reciprocal: $\left.\dfrac{df^{-1}}{dx}\right|_{f(a)} = \dfrac{1}{\left.\dfrac{df}{dx}\right|_{a}}$

## EXPLORATION 1 Finding a Derivative on an Inverse Graph Geometrically

Let $f(x) = x^5 + 2x - 1$. Since the point $(1, 2)$ is on the graph of $f$, it follows that the point $(2, 1)$ is on the graph of $f^{-1}$. Can you find

$a, f(a)$

$$\frac{\partial f}{\partial x} = 5x^4 + 2 \text{ at } a = 1 \quad = 7$$

$$\frac{df^{-1}}{dx}(2), = \frac{1}{7}$$

$$\left.\frac{\partial f^{-1}}{\partial x}\right|_{f(a)} = \frac{1}{\left.\frac{\partial f}{dx}\right|_a}$$

the value of $df^{-1}/dx$ at 2, without knowing a formula for $f^{-1}$?

1. Graph $f(x) = x^5 + 2x - 1$. A function must be one-to-one to have an inverse function. Is this function one-to-one?
2. Find $f'(x)$. How could this derivative help you to conclude that $f$ has an inverse?
3. Reflect the graph of $f$ across the line $y = x$ to obtain a graph of $f^{-1}$.
4. Sketch the tangent line to the graph of $f^{-1}$ at the point $(2, 1)$. Call it $L$.
5. Reflect the line $L$ across the line $y = x$. At what point is the reflection of $L$ tangent to the graph of $f$?
6. What is the slope of the reflection of $L$?
7. What is the slope of $L$?
8. What is $\dfrac{df^{-1}}{dx}(2)$?

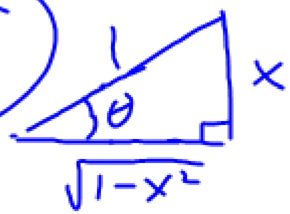
① Evaluate

ⓐ $\arcsin\left(-\frac{1}{2}\right)$ ⓑ $\arctan(\sqrt{3})$ ⓒ $\sin^{-1}(0.3)$

$-\frac{\pi}{6}$ rad. $\frac{\pi}{3}$ rad. $\approx 0.31$ rad.

② Find $\cos(y)$ if $y = \arcsin(x)$

$\cos\left(\sin^{-1}(x)\right)$ $\cos\theta = \sqrt{1-x^2}$




③ Solve for $x$, $\tan \arctan(2x-3) = \tan \frac{\pi}{4}$

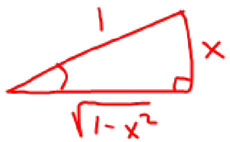
$$2x-3 = 1$$
$$+3 \quad +3$$
$$2x = 4$$
$$x = 2$$

$y = \sin^{-1}(x) \longrightarrow \sin(y) = x$

$-\frac{\pi}{2} < x < \frac{\pi}{2}$

$\frac{d}{dx}\sin(y) = \frac{d}{dx}x$

$\cos(y)\frac{dy}{dx} = 1$




$\cos(y) = \sqrt{1-\sin^2 y}$

$= \sqrt{1-\sin^2(\sin^{-1}(x))}$

$= \sqrt{1-x^2}$

$\frac{dy}{dx} = \frac{1}{\cos(y)} \Longrightarrow \frac{dy}{dx} = \frac{1}{\cos(\sin^{-1}(x))}$

$$\boxed{y = \sin^{-1}(x) \longrightarrow \frac{dy}{dx} = \frac{1}{\sqrt{1-x^2}}}$$

$$\boxed{y = \arcsin(u) \longrightarrow \frac{dy}{dx} = \frac{1}{\sqrt{1-u^2}}\cdot\frac{du}{dx}}$$

$f^{-1}(x)$ = inverse

$y = \tan^{-1}(x) \qquad \tan y = x$

$$\frac{d}{dx}\tan y = \frac{dx}{dx}$$

$$\sec^2(y)\frac{dy}{dx} = 1$$

$$\frac{dy}{dx} = \frac{1}{\sec^2(y)} \rightarrow \frac{dy}{dx} = \frac{1}{1+\tan^2 y}$$

$$= \frac{1}{1+\tan^2(\tan^{-1}(x))}$$

$$\boxed{y = \tan^{-1}(x) \longrightarrow \frac{dy}{dx} = \frac{1}{1+x^2}}$$

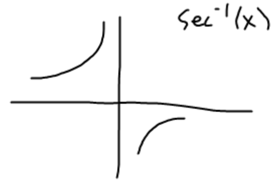
$$\boxed{y = \tan^{-1}(u) \longrightarrow \frac{dy}{dx} = \frac{1}{1+u^2}\cdot\frac{du}{dx}}$$

$y = \sec^{-1}(x) \qquad \sec(y) = x$

$$\frac{d}{dx}\sec(y) = \frac{d}{dx}x$$

$$\sec y \tan y \frac{dy}{dx} = 1$$

$$\frac{dy}{dx} = \frac{1}{\sec y \tan y} \qquad \tan y = \pm\sqrt{\sec^2 y - 1}$$




$$\frac{dy}{dx} = \pm \frac{1}{\sec y \sqrt{\sec^2 y - 1}}$$

$$\frac{dy}{dx} = \pm \frac{1}{\sec(\sec^{-1}(x))\sqrt{\sec^2(\sec^{-1}(x)) - 1}} \Rightarrow + \frac{1}{x\sqrt{x^2-1}}$$

$$\boxed{y = \sec^{-1}(x) \longrightarrow \frac{dy}{dx} = \frac{1}{|x|\sqrt{x^2-1}}}$$

$$\boxed{y = \sec^{-1}(u) = \frac{dy}{dx} = \frac{1}{|u|\sqrt{u^2-1}}}$$

Inverse cofunction Identities

$$\cos^{-1}(x) = \frac{\pi}{2} - \sin^{-1}(x)$$

$$\cot^{-1}(x) = \frac{\pi}{2} - \tan^{-1}(x)$$

$$\csc^{-1}(x) = \frac{\pi}{2} - \sec^{-1}(x)$$

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$$\frac{d}{dx}\cos^{-1}(x) = -\frac{d}{dx}\sin^{-1}(x) \Rightarrow \frac{d\cos^{-1}(x)}{dx} = -\frac{1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx}\cot^{-1}(x) = -\frac{d}{dx}\tan^{-1}(x) \Rightarrow -\frac{1}{1+x^2}$$

$$\frac{d}{dx}\csc^{-1}(x) = -\frac{d}{dx}\sec^{-1}(x) \Rightarrow -\frac{1}{|x|\sqrt{x^2-1}}$$

Sect. 3.8

#1-8 (3-4)

# 9-12 (1-2)

#13-22 (4-5)

#23-26 (1)

27-29 (2)